<table>
<thead>
<tr>
<th>Question</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$16,875.41</td>
</tr>
<tr>
<td>2</td>
<td>$175.87</td>
</tr>
<tr>
<td>3</td>
<td>$1155.51</td>
</tr>
<tr>
<td>4</td>
<td>$3900.55</td>
</tr>
<tr>
<td>5</td>
<td>$2497.90</td>
</tr>
<tr>
<td>6</td>
<td>$163.89</td>
</tr>
<tr>
<td>7</td>
<td>$813,057.86</td>
</tr>
<tr>
<td>8</td>
<td>$x = -3, y = 4</td>
</tr>
<tr>
<td>9</td>
<td>$x = y + 7, y = \text{ any real number}$</td>
</tr>
<tr>
<td>10</td>
<td>$\begin{bmatrix} -15 \ -15 \ -62 \end{bmatrix}$</td>
</tr>
<tr>
<td>11</td>
<td>$\begin{bmatrix} \frac{5}{9} &amp; \frac{1}{3} \ \frac{1}{3} &amp; 0 \end{bmatrix}$</td>
</tr>
<tr>
<td>12</td>
<td>$1791.08</td>
</tr>
<tr>
<td>13</td>
<td>$212,249.27$ or $212,249.28$</td>
</tr>
<tr>
<td>14</td>
<td>$1848.92$</td>
</tr>
</tbody>
</table>
1-11: Short answer questions. Each question is worth 5 points; there is no partial credit.

You must show your work, box your final answers and write your final answers on your Answer Page.

Answers without supporting work may be graded as incorrect.

1. If you had invested $14,000 on Jan 1, 2009, at 3% interest compounded quarterly, how much would you have on April 1, 2015? (Round your answer to the nearest cent.)

$16,875.41

2. An amount of $4000 is deposited into a savings account at 4% interest compounded quarterly. How much interest is earned during the third year (from the end of the second year to the end of the third year)? (Round your answer to the nearest cent.)

$175.87
3. During Jack’s first year at college, his father had been sending him $130 per month for incidental expenses. For the sophomore year, his father decided instead to make a deposit into a savings account on August 1 and have his son withdraw $130 on the first of each month from September 1 to May 1. If the bank pays 3% interest compounded monthly, how much should Jack’s father deposit? (Round your answer to the nearest cent.)

$1155.51

4. On her eighth birthday, a girl inherits $8000 which is to be used for her college education. The money is deposited into a trust fund that will pay her $R$ dollars on her 18th, 19th, 20th, and 21st birthdays. Find $R$ if the money earns 6% interest compounded annually. (Hint: Calculate the value of the trust fund, to the nearest cent, on the girl’s 17th birthday. Then use that amount to calculate $R$. Round your final answer to the nearest cent.)

$3900.55$
5. A car is purchased for $7472.39 with $3000 down and a loan to be repaid at $100 a month for 2 years followed by a balloon payment. If the interest rate is 6% compounded monthly, how large will the balloon payment be? (Round your answer to the nearest cent.)

$2497.90

6. Using the add-on method, what is the monthly payment for a $5000 loan at 6% interest for three years? (Round your answer to the nearest cent.)

$163.89
7. If a 20-year-old deposits $2000 each year into a traditional IRA for 50 years at 7% interest compounded annually, and then retires at age 70, how much money will be in the account upon retirement? (Round your answer to the nearest cent. Do NOT subtract for taxes.)

\[ \$813,057.86 \]

8. Use the Gauss-Jordan elimination method to find all solutions of the system of equations. If there are no solutions, write No Solutions.

\[
\begin{align*}
    x + 2y &= 5 \\
    5x + 6y &= 9
\end{align*}
\]

\[ x = -3, \ y = 4 \]
9. Use the Gauss-Jordan elimination method to find all solutions of the system of equations. If there are no solutions, write No Solutions.

\[
\begin{align*}
2x - 2y &= 14 \\
-6x + 6y &= -42 \\
x &= y + 7, \ y = \text{any real number}
\end{align*}
\]

10. Perform matrix multiplication, if possible. If the matrices cannot be multiplied, write Matrices Cannot Be Multiplied.

\[
\begin{bmatrix}
4 & 1 & 0 \\
4 & 0 & 9 \\
3 & -5 & 1
\end{bmatrix}
\begin{bmatrix}
-6 \\
9 \\
1
\end{bmatrix}
= 
\begin{bmatrix}
-15 \\
-15 \\
-62
\end{bmatrix}
\]
11. Find the inverse of the given matrix if an inverse exists. If no inverse exists, write No Inverse.

\[
\begin{bmatrix}
0 & -3 \\
3 & 5
\end{bmatrix}
\]

\[
\begin{bmatrix}
5 & 1 \\
9 & 3 \\
-1 & 0
\end{bmatrix}
\]

12-14: Workout questions. Each question is worth 5 points; partial credit is possible.

You must show your work, box your final answers and write your final answers on your Answer Page.

12. Consider a 20-year $250,000 5/1 ARM having a 2.4% margin and based on the CMT index. Suppose the interest rate is initially 6% and the value of the CMT is 4.1% five years later. Assume that all interest rates use monthly compounding. Calculate the monthly payment for the first 5 years. (Round your answer to the nearest cent.)

(Grading: 5 points total. If answer is incorrect, 1 point for each step.)

\[
i = \frac{0.06}{12} = 0.005
\]

\[
n = (12)(20) = 240
\]

\[
250,000 = \frac{1 - (1 + 0.005)^{240}}{0.005} \cdot R
\]

$1791.08
13. For the mortgage in Question 12, calculate the unpaid balance at the end of the first 5 years. (Round your answer to the nearest cent.)

(Grading: 5 points total. If answer is incorrect, 1 point for each step. 5 points for correct technique with incorrect inputs from Question 12.)

\[ i = \frac{0.06}{12} = 0.005 \]

\[ n = (12)(15) = 180 \]

\[ P = \frac{1 - (1 + 0.005)^{-180}}{0.005} \cdot (1791.08) \]

\$212,249.27 or \$212,249.28

---

14. For the mortgage in Question 12, use your rounded answer in Question 13 to calculate the monthly payment for the 6th year. (Round your answer to the nearest cent.)

(Grading: 5 points total. If answer is incorrect, 1 point for each step. 5 points for correct technique with incorrect inputs from Questions 12 and 13.)

\[ r = 4.1\% + 2.4\% = 6.5\% \]

\[ i = \frac{0.065}{12} \]

\[ n = (12)(15) = 180 \]

\[ 212,249.28 = \frac{1 - (1 + \frac{0.065}{12})^{-180}}{\frac{0.065}{12}} \cdot R \]

\$1848.92
Potentially Helpful Formulas

\[ F = (1 + i)^n P \]

\[ P = \frac{F}{(1 + i)^n} \]

\[ r_{\text{eff}} = \text{APY} = (1 + i)^m - 1 \]

\[ F = \frac{(1 + i)^n - 1}{i} \cdot R \]

\[ P = \frac{1 - (1 + i)^{-n}}{i} \cdot R \]

\[ R = \frac{P(1 + rt)}{12t} \]
<table>
<thead>
<tr>
<th>Question</th>
<th>Answer</th>
<th>Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$C = 50x + 40y$ or $C = 0.50x + 0.40y$</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>See exam booklet</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>See exam booklet</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>1 Can Brand A, 6 Cans Brand B</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>Cases $= 320x + 220y$</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>See exam booklet</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>See exam booklet</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>9 High Capacity, 21 Low Capacity</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>$C = 20x + 20y + 690$</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>See exam booklet</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>See exam booklet</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>$(2, 2), (4, 0)$ and $(3, 1)$</td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>${6}$</td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>See exam booklet</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>170</td>
<td></td>
</tr>
<tr>
<td></td>
<td><strong>Total</strong></td>
<td></td>
</tr>
</tbody>
</table>
Questions 1-4: Mr. Smith decides to feed his pet Doberman pinscher a combination of two dog foods. He is now considering a combination of two foods which are packaged in cans. Each can of brand A contains 3 units of protein, 1 unit of carbohydrates and 2 units of fat. A can of brand A costs 50 cents. Each can of brand B contains 1 unit of protein, 1 unit of carbohydrates and 4 units of fat. A can of brand B costs 40 cents.

Mr. Smith feels that each day his dog should have at least 9 units of protein, 7 units of carbohydrates and 24 units of fat. He would like to provide these nutritional requirements at minimum cost.

Let \( x \) be the number of cans of brand A and let \( y \) be the number of cans of brand B.

1. Write the objective equation for this problem.

(5 points. \( C \) is not necessary.)

\[
C = 50x + 40y \text{ or } C = 0.50x + 0.40y
\]

2. Write the five inequalities associated with this problem.

(5 points. 1 point for each inequality.)

\[
\begin{align*}
3x + y & \geq 9 \\
x + y & \geq 7 \\
2x + 4y & \geq 24 \\
x & \geq 0 \\
y & \geq 0
\end{align*}
\]
3. Graph your inequalities on the grid below. Label the feasible set.

(5 points. 1 point for each line, 1 point for correct feasible set position, 1 point for all correct.)

4. Use the vertices of the feasible set and the objective equation to find the number of cans of Brand A and Brand B that will meet Mr. Smith’s minimum nutritional requirements and minimize cost.

(5 points. 1 point for each vertex, 1 point for final answer.)

\[(0, 9) \rightarrow C = \$3.60\]
\[(1, 6) \rightarrow C = 0.50 + 2.40 = \$2.90\]
\[(2, 5) \rightarrow C = 1.00 + 2.00 = \$3.00\]
\[(12, 0) \rightarrow C = \$6.00\]

1 Can Brand A, 6 Cans Brand B
Questions 5-8: A company is buying new trucks and wishes to maximize the number of cases of merchandise shipped at one time.

The high-capacity trucks cost $50,000 and hold 320 cases of merchandise. The low-capacity trucks cost $30,000 and hold 220 cases of merchandise.

The company has budgeted $1,080,000 for the new trucks and has a maximum of 30 people qualified to drive the trucks. Due to availability limitations, the company can purchase at most 12 high-capacity trucks.

Let \( x \) be the number of high-capacity trucks and \( y \) be the number of low-capacity trucks to be purchased.

5. Write the objective equation for this problem.

(5 points. "Cases" is not necessary.)

\[
\text{Cases} = 320x + 220y
\]

6. Write the five inequalities associated with this problem.

(5 points. 1 point for each inequality.)

\[
50,000x + 30,000y \leq 1,080,000
\]

\[
x \leq 12
\]

\[
x + y \leq 30
\]

\[
x \geq 0
\]

\[
y \geq 0
\]
7. Graph your inequalities on the grid below. Label the feasible set.

(5 points. 1 point for each line, 1 point for correct feasible set position, 1 point for all correct.)

8. Use the vertices of the feasible set and the objective equation to find the number of high-capacity and low-capacity trucks that will maximize the number of cases of merchandise that can be delivered.

(5 points. 1 point for each vertex, 1 point for final answer.)

(0, 30) → Cases = 6600

(9, 21) → Cases = 2880 + 4620 = 7500

(12, 16) → Cases = 3840 + 3520 = 7360

(12, 0) → Cases = 3840

9 High Capacity, 21 Low Capacity
Questions 9-12: A foreign-car dealer with warehouses in New York and Baltimore receives orders from dealers in Philadelphia and Trenton.

The dealer in Philadelphia needs 2 cars and the dealer in Trenton needs 7 cars. The New York warehouse has 5 cars available and the warehouse in Baltimore has 8 cars available.

The shipping costs are as follows: $120 per car from Baltimore to Philadelphia; $90 per car from Baltimore to Trenton; $100 per car from New York to Philadelphia; $70 per car from New York to Trenton.

Let \((x, y)\) correspond to \(x\) cars shipped from Baltimore to Trenton and \(y\) cars shipped from Baltimore to Philadelphia. The dealer wishes to find the \((x, y)\) that minimizes the shipping costs.

9. Write the objective equation for this problem.

(5 points. \(C\) is not necessary.)

\[
C = 20x + 20y + 690 \text{ or } C = 90x + 120y + 70(7 - x) + 100(2 - y)
\]

10. Write the six inequalities associated with this problem.

(5 points. 0.5 points for \(x \geq 0\) and \(y \geq 0\), 1 point each for other inequalities.)

\[
\begin{align*}
x & \geq 0 \\
y & \geq 0 \\
7 - x & \geq 0 \\
2 - y & \geq 0 \\
x + y & \leq 8 \\
(7 - x) + (2 - y) & \leq 5
\end{align*}
\]
11. Graph your inequalities on the grid below. Label the feasible set.

(5 points. 1 point for each line, 1 point for correct feasible set position.)

![Graph of inequalities with feasible set labeled as FS.]

12. Use the vertices of the feasible set and the objective equation to find all valid values of $x$ and $y$ which will minimize the shipping costs.

(5 points. 0.5 points for each vertex, 1 point each for $\{(2, 2)\}$ and $\{(4, 0)\}$, 0.5 points for $\{(3, 1)\}$ or "line segment connecting $\{(2, 2)\}$ and $\{(4, 0)\}"")

\[
\begin{align*}
(2, 2) \rightarrow C &= 40 + 40 + 690 = 770 \\
(6, 2) \rightarrow C &= 120 + 40 + 690 = 850 \\
(7, 1) \rightarrow C &= 140 + 20 + 690 = 850 \\
(7, 0) \rightarrow C &= 140 + 690 = 830 \\
(4, 0) \rightarrow C &= 80 + 690 = 770
\end{align*}
\]

\((2, 2), (4, 0)\) and \((3, 1)\) (which is also on the line segment connecting these two points)
13-15: Short answer questions. Each question is worth 5 points; no partial credit.

You must show your work, box your final answers and write your final answers on your Answer Page.

Answers without supporting work may be graded as incorrect.

13. Let \( U = \{1, 2, 3, 4, 5, 6\} \), \( A = \{1, 2, 3\} \), \( B = \{1, 3, 5\} \), and \( C = \{4, 5\} \).

List the elements in the following set: \( A' \cap C' \)

\[ \{6\} \]

14. In the Venn diagram below, shade in the following region: \( A \cap (B \cup C) \)
15. A merchant surveyed 370 people to determine the way they learned about an upcoming sale. The survey showed that 110 learned about the sale from the radio, 200 from television, 160 from the newspaper, 40 from radio and television, 30 from radio and newspapers, 90 from television and newspapers, and 20 from all three sources. How many people learned of the sale from radio or television but not the newspaper?

170
<table>
<thead>
<tr>
<th>Question</th>
<th>Answer</th>
<th>Score</th>
</tr>
</thead>
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<tr>
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<td>28</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>90,000</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>360,360</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>1665</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>150</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>0.3</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>7 to 18</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>0.1181</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>$\frac{9}{10}$</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>0.5391</td>
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<tr>
<td>11</td>
<td>$\frac{1}{6}$</td>
<td></td>
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<tr>
<td>12</td>
<td>1 to 139,150,439</td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>See exam booklet</td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>$\frac{5}{24}$, $\frac{1}{6}$, $\frac{3}{8}$</td>
<td></td>
</tr>
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<td>15</td>
<td>$\frac{5}{9}$</td>
<td></td>
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<tr>
<td>Total</td>
<td></td>
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</tr>
</tbody>
</table>
1. A football squad can elect a captain and an assistant captain in 756 possible ways. How many members does the squad have?

28

2. A number is said to be a palindrome if it reads the same backwards as forwards (for example, 54845). How many 10-digit numbers are palindromes?

90,000
3. At a benefit concert, fifteen bands have volunteered to perform but there is only enough time for five of the bands to play. How many lineups are possible? (Please note that the order in which the bands perform matters in this problem.)

360,360

4. A class has 17 students, of which 4 are seniors. How many committees of size 4 can be selected if at least one member of each committee must be a senior?

1665
5. Refer to the map in the figure below. How many of the routes from A to B (with no backtracking) pass through the point C?

6. Table 1 below summarizes the age distribution for a company’s employees. Each probability is the likelihood that a randomly selected employee is in the specified age group. What is the probability that an employee selected at random is at least 50 years old?

<table>
<thead>
<tr>
<th>Age(years)</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>20-34</td>
<td>0.35</td>
</tr>
<tr>
<td>20-49</td>
<td>0.7</td>
</tr>
<tr>
<td>20-64</td>
<td>0.9</td>
</tr>
<tr>
<td>20-79</td>
<td>1</td>
</tr>
</tbody>
</table>

0.3
7. For a certain salesman, the probability of selling a car today is 0.28. Find the odds in favor of him selling a car today.

7 to 18

8. An urn contains six green balls and ten white balls. A sample of four balls is selected at random from the urn. Find the probability that the sample contains more green balls than white balls. (Express your answer as a decimal rounded to four decimal places.)

0.1181
9. A factory produces fuses, which are packaged in boxes of 16. Three fuses are selected at random from each box for inspection. The box is rejected if at least one of these three fuses is defective. What is the probability that a box containing eight defective fuses will be rejected? (Express your answer as a fraction.)

\[
\frac{9}{10}
\]

10. An airport limousine has four passengers and stops at nine different hotels. What is the probability that two or more people will be staying at the same hotel? (Assume that each person is just as likely to stay in one hotel as another. Express your answer as a decimal rounded to four decimal places.)

\[
0.5391
\]
11. A man, a woman, and their ten children randomly stand in a row for a family picture. What is the probability that the parents will be standing next to each other? (Express your answer as a fraction.)

\[ \frac{1}{6} \]

12. The winner of the Superball lottery must correctly pick a set of 5 numbers from 1 through 54 (order does not matter) and then correctly pick one number (called the superball) from 1 to 44. What are the odds of winning the Superball lottery?

1 to 139,150,439
Questions 13-15: Five ordinary quarters and a fake quarter with two heads are placed in a hat. One quarter is selected at random and tossed twice.

13. Draw a tree diagram for this problem. Label your outcomes for the selection from the hat as "Ordinary" and "Fake". Label your outcomes for each of the two tosses as "H" and "T". Write the probabilities for these outcomes on the branches of your tree.

(5 points. 0.5 points for each branch.)

14. Calculate the following probabilities: \( \Pr(\text{Ordinary} \cap \text{HH}) \), \( \Pr(\text{Fake} \cap \text{HH}) \), \( \Pr(\text{HH}) \)

(5 points. 2 points for first two probabilities, 1 point for the third. Full credit for correct technique and arithmetic with bad inputs from question 13.)

\[
\begin{align*}
\Pr(\text{Ordinary} \cap \text{HH}) &= \frac{5}{24} \\
\Pr(\text{Fake} \cap \text{HH}) &= \frac{1}{6} \\
\Pr(\text{HH}) &= \frac{3}{8}
\end{align*}
\]
15. If the outcome is "HH", what is the probability that the ordinary quarter was selected? Use a conditional probability calculation to solve this problem.

(5 points. 3 points for correct conditional probability formula, 2 points for correct calculation. Full credit for correct technique and arithmetic with bad inputs from questions 13 and 14.)

\[
\Pr(\text{Ordinary} \mid \text{HH}) = \frac{\Pr(\text{Ordinary} \cap \text{HH})}{\Pr(\text{HH})} = \frac{5}{9}
\]
1. Solve the following system of equations using the Gauss-Jordan elimination method.

\[
\begin{align*}
3x + 9y &= 3 \\
-6x - 18y &= -6
\end{align*}
\]

Select the correct choice below and fill in any answer boxes within your choice.

- **A.** There is one solution. The solution is \(x = \) _________ and \(y = \) _________.
  (Simplify your answer.)

- **B.** There are infinitely many solutions. If \(y\) is any real number, \(x = \) _________.
  (Type an expression using \(y\) as the variable.)

- **C.** There is no solution.

2. Use the Gauss-Jordan elimination method to find all solutions of the system of equations.

\[
\begin{align*}
x + 5y &= -3 \\
-3x + 5y &= -11 \\
-x + 3y &= -5
\end{align*}
\]

Select the correct choice below and fill in any answer boxes within your choice.

- **A.** There is one solution. The solution is \(x = \) _________ and \(y = \) _________.
  (Simplify your answers.)

- **B.** There are infinitely many solutions. If \(y\) is any real number, \(x = \) _________.
  (Type an expression using \(y\) as the variable.)

- **C.** There is no solution.

3. Use Gauss-Jordan elimination to find the solution to the following system of equations.

\[
\begin{align*}
-x + y + z &= -2 \\
-x + 4y - 8z &= -17 \\
5x - 3y - 11z &= 0
\end{align*}
\]

Select the correct choice below and fill in any answer boxes within your choice.

- **A.** There is one solution. The solution is \(x = \) _________, \(y = \) _________, and \(z = \) _________.
  (Simplify your answer.)

- **B.** There are infinitely many solutions. If \(z\) is any real number, \(x = \) _________ and \(y = \) _________.
  (Type an expression using \(z\) as the variable.)

- **C.** There is no solution.
4. Solve the following system of equations using the Gauss-Jordan elimination method.

\[
\begin{align*}
  x - 3y + 3z &= -10 \\
 -2x + 6y - 6z &= 19 \\
 -x + 3y - 3z &= 8
\end{align*}
\]

Select the correct choice below and fill in any answer boxes within your choice.

A. There is one solution. The solution is \( x = \) ________, \( y = \) ________, and \( z = \) ________.  
\( \text{(Simplify your answer.)} \)

B. There are infinitely many solutions. If \( z \) is any real number, \( x = \) ________ and \( y = \) ________.

C. There is no solution.

5. Perform the multiplication.

\[
\begin{pmatrix}
0.5 & 0.4 \\
0.3 & 0.1
\end{pmatrix}
\begin{pmatrix}
0.5 & 0.4 \\
0.3 & 0.1
\end{pmatrix}
\]

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

A. \[
\begin{pmatrix}
0.5 & 0.4 \\
0.3 & 0.1
\end{pmatrix}
\begin{pmatrix}
0.5 & 0.4 \\
0.3 & 0.1
\end{pmatrix}
= \]
\( \text{__________} \)  
\( \text{(Type an integer or decimal for each matrix element.)} \)

B. The product is undefined.

6. In a certain town, the proportions of voters voting Democratic and Republican by various age groups is summarized by matrix \( A \), and the population of voters in the town by age group is given by matrix \( B \).

Interpret the entries of the matrix product \( BA \).

\[
\begin{pmatrix}
0.71 & 0.29 \\
0.51 & 0.49 \\
0.35 & 0.65
\end{pmatrix}
= A
\]

\[
\begin{pmatrix}
3000 & 7000 & 9000 \\
30 & 30 & 30
\end{pmatrix}
= B
\]

In the matrix \( BA \), the first entry means that there are \( \text{__________} \) voters \( \text{(1) ____________} \) and the second entry means that there are \( \text{__________} \) voters \( \text{(2) ____________} \).

(1) \( \text{__________} \) over 50 \( \text{__________} \) under 30 \( \text{__________} \) between 30 and 50
(2) \( \text{__________} \) over 50 \( \text{__________} \) under 30
7. Find the inverse of the given matrix.

\[
\begin{bmatrix}
0.3 & 0.1 \\
0.7 & 0.9
\end{bmatrix}
\]

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- **A.** The matrix is not invertible.
- **B.** The inverse of the given matrix is _______.
  (Type an integer or decimal for each matrix element.)

8. It is found that the number of married and single adults in a certain town are subject to the following statistics. Suppose that \(x\) and \(y\) denote the number of married and single adults, respectively, in a given year (say as of January 1) and let \(m\) and \(s\) denote the corresponding numbers for the following year. Complete parts \(a\) through \(d\).

\[
\begin{align*}
0.6x + 0.4y &= m \\
0.1x + 0.9y &= s
\end{align*}
\]

(a) Write this system of equations in matrix form.

\[
\begin{bmatrix}
x \\
y
\end{bmatrix}
= 
\begin{bmatrix}
m \\
s
\end{bmatrix}
\]

(Type an integer or decimal for each matrix element.)

(b) Solve the resulting matrix equation for \(X = \begin{bmatrix} x \\ y \end{bmatrix}\).

\[
\begin{bmatrix}
x \\
y
\end{bmatrix}
= 
\begin{bmatrix}
m \\
s
\end{bmatrix}
\]

(Type an integer or decimal for each matrix element.)

(c) Suppose that in a given year there were found to be 100,000 married adults and 50,000 single adults. How many married (respectively, single) adults were there the preceding year?

In the preceding year, there were _________ married adults and _________ single adults.

(d) How many married (respectively, single) adults were there two years ago?

Two years ago, there were _________ married adults and _________ single adults.

9. A nutritionist, working for NASA, must meet certain minimum nutritional requirements and yet keep the weight of the food at a minimum. He is considering a combination of two foods, which are packaged in tubes. Each tube of food A contains 4 units of protein, 2 units of carbohydrates, and 2 units of fat and weighs 3 pounds. Each tube of food B contains 3 units of protein, 6 units of carbohydrates, and 1 unit of fat and weighs 2 pounds. The requirement calls for 48 units of protein, 42 units of carbohydrates, and 18 units of fat. How many tubes of each food should be supplied to the astronauts?

The number of tubes of food A is _________.

The number of tubes of food B is _________.
10. Mr. Smith decides to feed his pet Doberman pinscher a combination of two dog foods. Each can of brand A contains 3 units of protein, 1 unit of carbohydrates, and 2 units of fat and costs 60 cents. Each can of brand B contains 1 unit of protein, 1 unit of carbohydrates, and 4 units of fat and costs 40 cents. Mr. Smith feels that each day his dog should have at least 5 units of protein, 3 units of carbohydrates, and 8 units of fat. How many cans of each dog food should he give to his dog each day to provide the minimum requirements at the least cost?

Mr. Smith should give his dog _____________ can(s) of brand A and _____________ can(s) of brand B to provide the minimum requirements at the least cost.

11. Mr. Jones has $13,000 to invest in three types of stocks, low-risk, medium-risk, and high-risk. He invests according to three principles. The amount invested in low-risk stocks will be at most $3,000 more than the amount invested in medium-risk stocks. At least $7,000 will be invested in low- and medium-risk stocks. No more than $10,000 will be invested in medium- and high-risk stocks. The expected yields are 6% for low-risk stocks, 7% for medium-risk stocks, and 8% for high-risk stocks. How much money should Mr. Jones invest in each type of stock to maximize his total expected yield?

Mr. Jones should invest $ _____________ in low-risk stocks, $ _____________ in medium-risk stocks, and $ _____________ in high-risk stocks.

12. A foreign-car dealer with warehouses in New York and Baltimore receives orders from dealers in Philadelphia and Trenton. The dealer in Philadelphia needs 2 cars and the dealer in Trenton needs 7. The New York warehouse has 3 cars and the Baltimore warehouse has 8. The cost of shipping cars from Baltimore to Philadelphia is $120 per car, from Baltimore to Trenton $90 per car, from New York to Philadelphia $100 per car, and from New York to Trenton $70 per car. Find the number of cars to be shipped from each warehouse to each dealer to minimize the shipping cost.

Let (x, y) correspond to x cars shipped from Baltimore to Trenton, y cars shipped from Baltimore to Philadelphia. Find (x, y). Choose the correct answer below.

- **A.** The minimum cost is achieved at (4,2).
- **B.** The minimum cost is achieved at (4,2), (6,0), or anywhere on the line segment connecting these two points.
- **C.** The minimum cost is achieved at (6,0).
- **D.** The minimum cost is achieved at (4,2) and (6,0).

13. A shipping company is buying new trucks. The high-capacity trucks cost $50,000 and hold 350 cases of merchandise. The low-capacity trucks cost $30,000 and hold 220 cases of merchandise. The company has budgeted $1,080,000 for the new trucks and has a maximum of 30 people qualified to drive the trucks. Due to availability limitations, the company can purchase at most 12 high-capacity trucks. How many of each type of truck should the company purchase to maximize the number of cases shipped at one time?

To maximize the number of cases of merchandise that can be shipped simultaneously, the company should purchase _____________ high-capacity trucks and _____________ low-capacity trucks.
14. Draw a three-circle Venn diagram and shade the portion corresponding to the set.

\[ T \cap (R \cup S) \]

Choose the correct diagram below.

A. 

B. 

C. 

D. 

15. An electronics store receives a shipment of 40 graphing calculators, including 8 that are defective. Four of the calculators are selected to be sent to a local high school.

A. How many selections can be made using the original shipment?

B. How many of these selections will contain no defective calculators?

16. Refer to the map in the figure below. How many of the routes from A to B pass through the point C?

How many routes are there from A to B?

17. An urn contains 11 numbered balls, of which 7 are red and 4 are white. A sample of 4 balls is to be selected. Complete parts (a) through (d).

(a) How many different samples are possible?

(b) How many samples contain all red balls?

(c) How many samples contain 2 red balls and 2 white balls?

(d) How many samples contain at least 3 red balls?
18. A bag of 9 apples contains 2 rotten apples and 7 good apples. A shopper selects a sample of 4 apples from the bag.

(a) How many different samples are possible?

(b) How many samples contain all good apples?

(c) How many samples contain at least 1 rotten apple?

19. How many different committees can be formed from 12 teachers and 44 students if the committee consists of 3 teachers and 2 students?

In how many ways can the committee of 5 members be selected?

20. If a "word" is interpreted to be a sequence of letters, how many five-letter words with no repeated letters contain two vowels? (Note that y is not considered a vowel.)

There are ________ words. (Simplify your answer.)

21. An urn contains eight green balls and five white balls. A sample of four balls is selected at random from the urn.

(a) Find the probability that the four balls have the same color.

(b) Find the probability that the sample contains more green balls than white balls.

(a) The probability that the four balls have the same color is ________.
(Type an integer or a simplified fraction.)

(b) The probability that the sample contains more green balls than white balls is ________.
(Type an integer or a simplified fraction.)

22. A factory produces fuses, which are packaged in boxes of 10. Three fuses are selected at random from each box for inspection. The box is rejected if at least one of these three fuses is defective. What is the probability that a box containing four defective fuses will be rejected?

The probability that the box containing four defective fuses will be rejected is ________.
(Type an integer or a simplified fraction.)

23. A man, a woman, and their ten children randomly stand in a row for a family picture. What is the probability that the parents will be standing next to each other?

The probability that the parents will be standing next to each other is ________.
(Type an integer or a simplified fraction.)
24. Let \( S \) be a sample space and \( E \) and \( F \) be events associated with \( S \). Suppose that \( \Pr(E) = 0.7 \), \( \Pr(F) = 0.3 \) and \( \Pr(E \cap F) = 0.2 \). Calculate the following probabilities.

\[
\begin{align*}
\text{a. } & \Pr(E|F) & & \text{c. } & \Pr(E'|F) \\
\text{b. } & \Pr(F|E) & & \text{d. } & \Pr(E'|F')
\end{align*}
\]

\[
\begin{align*}
\text{a. } & \Pr(E|F) = & & \text{c. } & \Pr(E'|F) = \\
\text{b. } & \Pr(F|E) = & & \text{d. } & \Pr(E'|F') = \text{ (Type an integer or a simplified fraction.)}
\end{align*}
\]

25. A bag contains seven red marbles and ten white marbles. If a sample of four marbles contains at least one white marble, what is the probability that all the marbles in the sample are white?

The probability is \( \boxed{\text{.}} \). (Round to four decimal places as needed.)

26. Suppose that we have a white urn containing five white balls and one red ball and we have a red urn containing one white ball and two red balls. An experiment consists of selecting at random a ball from the white urn and then (without replacing the first ball) selecting at random a ball from the urn having the color of the first ball. Find the probability that the second ball is red.

The probability that the second ball is red is \( \boxed{\text{.}} \).

27. Four ordinary quarters and a fake quarter with two heads are placed in a hat. One quarter is selected at random and tossed twice. If the outcome is "HH," what is the probability that the fake quarter was selected?

\[
\Pr(\text{fake}|HH) = \frac{\text{ }}{\text{}} \text{ (Type an integer or a simplified fraction.)}
\]

28. Suppose 500 athletes are tested for a drug, one in twenty-five has used the drug, the test has a 98% specificity and the test has a 100% sensitivity. That is, the probability of a false positive is 2% and there is no chance that the user of the drug will go undetected. If an athlete in the group tests positive, what is the probability that he or she has used the drug?

\[
\Pr(\text{used}|\text{POS}) = \frac{\text{ }}{\text{}} \text{ (Type an integer or decimal rounded to the nearest hundredth as needed.)}
\]
29. Find the five-number summary and the interquartile range for the given set of numbers, and then draw the box plot.

10, 12, 13, 15, 16, 17, 19, 20, 21, 22, 23

The five-number summary:

\[
\text{min} = \underline{10}, \quad Q_1 = \underline{12}, \quad Q_2 = \underline{17}, \quad Q_3 = \underline{20}, \quad \text{max} = \underline{23}.
\]

The interquartile range is \( \underline{9} \).

Which graph is representative of the above data?

- [ ] A.
- [ ] B.
- [ ] C.
- [ ] D.

30. An urn contains 2 red balls and 9 white balls. A sample of 2 balls is selected at random and the number of red balls observed. Determine the probability distribution for this experiment and draw its histogram.

The probability to draw zero red balls is \( \underline{\frac{1}{5}} \), to draw one red ball is \( \underline{\frac{3}{10}} \), and to draw two red balls is \( \underline{\frac{1}{10}} \). (Simplify your answers.)

Choose the correct histogram below.

- [ ] A.
- [ ] B.
- [ ] C.
- [ ] D.

31. In a certain carnival game the player selects two balls at random from an urn containing 4 red balls and 6 white balls. The player receives $5 if he draws two red balls and $2 if he draws one red ball. He loses $1 if no red balls are in the sample. Determine the probability distribution for the experiment of playing the game and observing the player's earnings.

The probability to draw two red balls is \( \underline{\frac{6}{35}} \), to draw one red ball is \( \underline{\frac{12}{35}} \), and to draw zero red balls is \( \underline{\frac{5}{7}} \). (Simplify your answers.)

32. A coin is tossed 10 times. Find the probability that the number of heads is exactly five.

The probability that the number of heads is exactly five is \( \underline{\frac{252}{1024}} \). (Round to four decimal places as needed.)

33. A coin is tossed 11 times. Find the probability of tossing five or six heads.

The probability of tossing five or six heads is \( \underline{\frac{21}{64}} \). (Round to four decimal places as needed.)
34. A single die is rolled seven times. Find the probability that 4 appears at most three times.

The probability that the number 4 appears at most three times is _________.
(Round to four decimal places as needed.)

35. Thirteen percent of U.S. residents are in their thirties. Consider a group of six U.S. residents selected at random. Find the probability that two or three of the people in the group are in their thirties.

The probability that two or three of the people in the group are in their thirties is _________.
(Round to four decimal places as needed.)

36. The table gives the relative frequency of the number of cavities for two groups of children trying different brands of toothpaste. Calculate the sample means to determine which group had fewer cavities.

<table>
<thead>
<tr>
<th>Number of cavities</th>
<th>Relative frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Group A</td>
</tr>
<tr>
<td>0</td>
<td>0.3</td>
</tr>
<tr>
<td>1</td>
<td>0.3</td>
</tr>
<tr>
<td>2</td>
<td>0.2</td>
</tr>
<tr>
<td>3</td>
<td>0.1</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>0.1</td>
</tr>
</tbody>
</table>

Find the sample mean for group A.

Find the sample mean for group B.

Determine which group had fewer cavities.

○ Group B
○ Group A

37. In a carnival game, the player selects two coins from a bag containing three silver dollars and six slugs. (Slugs are fake, worthless coins that look like real coins.) Write down the probability distribution for the winnings and determine how much the player would have to pay so that he would break even, on the average, over many repetitions of the game.

The player should pay _________ ¢ per play to break even.
(Round to the nearest cent as needed.)

38. The promoter of a football game is concerned that it will rain. She has the option of spending $11,700 on insurance that will pay $39,000 if it rains. She estimates that the revenue from the game will be $61,700 if it does not rain and $26,700 if it does rain. What must the chance of rain be if she is ambivalent about this insurance?

Choose the correct answer below.

○ 15%
○ 30%
○ 45%
○ 29%

39. Compute the variance of the probability distribution in the table below.

<table>
<thead>
<tr>
<th>Outcome</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>65</td>
<td>0.5</td>
</tr>
<tr>
<td>66</td>
<td>0.1</td>
</tr>
<tr>
<td>67</td>
<td>0.1</td>
</tr>
<tr>
<td>68</td>
<td>0.3</td>
</tr>
</tbody>
</table>

\( \sigma^2 = \___________ \) (Type an integer or a decimal.)
40. A manufacturer produces widgets that are packaged in boxes of 200. The probability of a widget being defective is 0.025. Find the mean and standard deviation for the number of defective widgets in a box.

The mean, \( \mu \), is \___________. (Round to the nearest hundredth as needed.)

The standard deviation, \( \sigma \), is \___________. (Round to the nearest thousandth as needed.)

41. For a certain type of light, the number of hours a bulb will burn before requiring replacement has a mean of 3000 hours and a standard deviation of 300 hours. Suppose that 6000 such bulbs are installed in an office building. Estimate the number that will require replacement between 2100 and 3900 hours from the time of installation.

At least \__________ bulbs will require replacement between 2100 and 3900 hours from the time of installation. (Round to the nearest whole number as needed.)

42. At the end of each month, for two years, $8,000 will be withdrawn from a savings account paying 2.1% interest compounded monthly. Calculate the present value of this decreasing annuity.

The present value is $\___________.
(Do not round until the final answer. Then round to the nearest cent as needed.)

43. At the end of each month, $100 is deposited into a savings account paying 2.7% interest compounded monthly. The balance after 7 years will be $9,234.87. What is the amount of interest earned?

The amount of interest earned is $\___________.
(Round to the nearest cent as needed.)

44. Consider a $66,717, 30-year mortgage at interest rate 6% compounded monthly with a $400 monthly payment.

(a) How much interest is paid the first month?
(b) How much of the first month's payment is applied to paying off the principal?
(c) What is the unpaid balance at the end of 25 years?
(d) How much interest is paid during the 301st month?

(a) The interest paid the first month is $\___________.
(Round to the nearest cent as needed.)

(b) $\__________ of the first month's payment is applied to paying off the principal.
(Round to the nearest cent as needed.)

(c) The unpaid balance at the end of 25 years is $\___________.
(Do not round until the final answer. Then round to two decimal places as needed.)

(d) The interest paid during the 301st month is $\___________.
(Round to the nearest cent as needed.)
45. A car manufacturer is offering the choice of a 0.3% loan compounded monthly for 36 months or $100 cash back on the purchase of a $19,500 new car. Complete parts (a) through (c) below.

(a) If a car buyer takes the 0.3% loan offer, how much will the monthly payment be?

The car buyer's monthly payment will be $\text{______________}.
(Do not round until the final answer. Then round to the nearest cent as needed.)

(b) If the car buyer takes the $100 cash-back offer and can borrow money from a local bank at 6% interest compounded monthly for three years, how much will the monthly payment be?

The car buyer's monthly payment will be $\text{______________}.
(Do not round until the final answer. Then round to the nearest cent as needed.)

(c) Which of the two offers is more favorable for the car buyer?

- A. The $100 cash-back offer is more favorable for the car buyer because the monthly payment is lower than for the 0.3% loan offer.
- B. The 0.3% loan is more favorable for the car buyer because the monthly payment is lower than for the $100 cash-back offer.
- C. The $100 cash-back offer is more favorable for the car buyer because the monthly payment is higher than for the 0.3% loan offer.
- D. The 0.3% loan offer is more favorable for the car buyer because the monthly payment is higher than for the $100 cash-back offer.
- E. Neither offer is more favorable than the other for the car buyer.

46. A loan of $105,494.20 is to be amortized over a 5-year term at 6% interest compounded monthly with monthly payments and a $30,000 balloon payment at the end of the term. Calculate the monthly payment.

The monthly payment is $\text{______________}.
(Do not round until the final answer. Then round to two decimal places as needed.)

47. If someone is 21 years old, deposits $1000 each year into a traditional IRA for 49 years at 7% interest compounded annually, and retires at age 70, how much money will be in the account upon retirement?

The future value of the traditional IRA is $\text{______________}.
(Round to the nearest cent as needed.)

48. Use the add-on method to determine the monthly payment for a $2,000 loan at 10% interest for one year.

The monthly payment is $\text{______________}.
(Round to the nearest cent as needed.)

49. Consider a 25-year mortgage of $300,000 at 6.0% interest compounded monthly where the loan is interest only for ten years. What is the monthly payment during the first ten years? last fifteen years?

The monthly payment for the first ten years is $\text{______________}.
(Round to the nearest cent as needed.)

The monthly payment for the last fifteen years is $\text{______________}.
(Round to the nearest cent as needed.)
50. Consider a 15-year $300,000 5/1 ARM having a 2.6% margin and based on the CMT index. Suppose the interest rate is initially 6% and the value of the CMT is 4.5% five years later. Assume that all interest rates use monthly compounding.
(a) Calculate the monthly payment for the first 5 years.
(b) Calculate the unpaid balance at the end of the first 5 years.
(c) Calculate the monthly payment for the 6th year.

(a) The monthly payment for the first five years is $\_\_\_\_\_\_\_.
(Round to the nearest cent as needed.)

(b) The unpaid balance after 5 years is $\_\_\_\_\_\_\_\_.
(Round to the nearest cent as needed.)

(c) The monthly payment for the 6th year is $\_\_\_\_\_\_\_\_.
(Round to the nearest cent as needed.)