INSTRUCTIONS

1. DO NOT OPEN THIS EXAM UNTIL INSTRUCTED TO BY YOUR ROOM LEADER. All exam pages must remain stapled. Do not separate or remove any pages. You will have 60 minutes to complete this exam.

2. This exam has 10 pages, including the cover sheet. There are 15 multiple choice questions. The answers on your scantron are your FINAL answers. If you change an answer, erase your old answer thoroughly. Only final answers on your scantron will be graded.

3. Some potentially helpful formulas can be found on page 10.
MULTIPLE CHOICE: Mark your FINAL answers on your scantron.

1. An amount of $600 is deposited into a savings account at 3% interest compounded monthly. How much interest is earned during the first five years? (Answers are rounded to the nearest cent.)
   a. $1924.96
   b. $96.97
   c. $2934.96
   d. $696.97
   e. $118.14

2. Consider a $250,187.42, 30-year mortgage at interest rate 6% compounded monthly with a $1,500 monthly payment. What is the unpaid balance at the end of 25 years? (Answers are rounded to the nearest cent.)
   a. $4836.82
   b. $232,810.30
   c. $77,588.34
   d. $24,242.14
   e. $25,000.00
3. A single deposit of $5000 is invested into a savings account earning 6% interest compounded semi-annually. If the initial deposit was on January 1, 2017, how much money will be in the account on July 1, 2022? (Answers are rounded to the nearest cent.)

   a. $5281.98
   b. $9491.49
   c. $6719.58
   d. $8954.24
   e. $6921.17

4. If someone 20 years old deposits $5000 each year into a savings account for 50 years at 8% interest compounded annually, how much money will be in the account when this person retires at age 70? (Answers are rounded to the nearest cent.)

   a. $61,167.42
   b. $2,868,850.78
   c. $736,079.71
   d. $228,809.82
   e. None of the above
5. A loan of $180,250.31 is to be amortized over a 6-year term at 6% interest compounded monthly with monthly payments and a $50,000 balloon payment at the end of the term. What is the monthly payment for this loan? (Answers are rounded to the nearest cent.)

   a) $2987.27
   b) $578.64
   c) $2697.94
   d) $2408.62
   e) None of the above.
6. A bank offers two accounts, Account A, which is 9.125% compounded monthly or Account B, which is 9.250% compounded quarterly. Calculate the APY for these accounts and determine which account is the better offer. (Answers are expressed as percentages and rounded to 3 places.)

- Account A 9.516%, Account B 9.576% (B is better)
- Account A 1.410%, Account B 1.298% (B is better)
- Account A 9.516%, Account B 9.576% (A is better)
- Account A 1.410%, Account B 1.298% (A is better)
- None of the above.

7. Using the add-on method, what is the monthly payment for a $3000 loan at 9% interest for two years? (Answers are rounded to the nearest cent.)

- $137.05
- $136.25
- $147.50
- $295.00
- None of the above
8. Consider a 15-year $100,000 5/1 ARM having a 3% margin and based on the CMT index. Suppose the interest rate is initially 6% and the value of the CMT is 5.1% five years later. Assume that all interest rates use monthly compounding. What is the monthly payment for the first 5 years? (Answers are rounded to the nearest cent.)

- a. $843.86
- b. $599.55
- c. $1055.56
- d. $961.43
- e. $500.00

9. For the mortgage in question 8, what is the monthly payment for the 6th year? (Answers are rounded to the nearest cent.)

- a. $1218.57
- b. $926.22
- c. $843.86
- d. $1055.27
- e. $961.43
10. Use the Gauss-Jordan elimination method to find all solutions of the system of equations:
\[
\begin{align*}
  x + 2y + 3z &= 5 \\
 2x + 3y - 4z &= 2
\end{align*}
\]

(a) \( x = -11, \ y = 8, \text{ and } z = 0 \)

(b) \( x = -17z - 11, \ y = 10z + 8, \text{ and } z = \text{ any real number} \)

(c) \( x = 17z - 11, \ y = -10z + 8, \text{ and } z = \text{ any real number} \)

(d) \( x = -11, \ y = 8, \text{ and } z = -7 \)

(e) There are no solutions.

11. Find the inverse (if it exists) of the matrix \( A = \begin{bmatrix} 4 & 2 \\ 6 & 3 \end{bmatrix} \).

(a) \( \begin{bmatrix} 4 & -2 \\ -6 & 3 \end{bmatrix} \)

(b) \( \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \)

(c) \( \begin{bmatrix} 4 & 2 \\ 6 & 3 \end{bmatrix} \)

(d) \( \begin{bmatrix} 3 & -2 \\ -6 & 4 \end{bmatrix} \)

(e) Matrix \( A \) does not have an inverse.
12. Use the Gauss-Jordan elimination method to find all solutions of the system of equations:

\[
\begin{align*}
\begin{cases}
  x + y + z &= 2 \\
  y + 2z &= 4 \\
  -x + 2y + z &= 2
\end{cases}
\end{align*}
\]

\(\text{a}\) \(x = 2,\ y = 4,\ \text{and}\ z = 2\)

\(\text{b}\) \(x = 0,\ y = 0,\ \text{and}\ z = 2\)

\(\text{c}\) \(x = 2,\ y = 0,\ \text{and}\ z = 0\)

\(\text{d}\) \(x = \text{any real number},\ y = \text{any real number},\ \text{and}\ z = 2\)

\(\text{e}\) \(\text{There are no solutions.}\)

13. Use the Gauss-Jordan elimination method to find all solutions for the following system of equations:

\[
\begin{align*}
\begin{cases}
  2x + 2y &= 4 \\
  3x + 9y &= 18
\end{cases}
\end{align*}
\]

\(\text{a}\) \(x = 4 - 2y,\ y = \text{any real number}\)

\(\text{b}\) \(x = \text{any real number},\ y = 2\)

\(\text{c}\) \(x = 2,\ y = 0\)

\(\text{d}\) \(x = 0,\ y = 2\)

\(\text{e}\) \(\text{There are no solutions.}\)
Questions 14-15: It is found that the number of married and single adults in a certain town are subject to the following statistics:

\[
\begin{align*}
0.8x + 0.6y &= m \\
0.1x + 0.7y &= s
\end{align*}
\]

where \( x \) and \( y \) denote the number of married and single adults, respectively, in a given year, and \( m \) and \( s \) denote the corresponding numbers for the following year.

Suppose that this year there were 100,000 married adults and 50,000 single adults.

14. How many married adults were there last year?
   - a) 110,000
   - b) 60,000
   - c) 80,000
   - d) 40,000
   - e) None of the above

15. How many single adults will there be next year?
   - a) 110,000
   - b) 50,000
   - c) 75,000
   - d) 100,000
   - e) None of the above
Potentially Helpful Formulas

\[ F = (1 + i)^n \cdot P \]
\[ P = \frac{F}{(1 + i)^n} \]
\[ r_{\text{eff}} = \text{APY} = (1 + i)^m - 1 \]
\[ F = \frac{(1 + i)^n - 1}{i} \cdot R \]
\[ P = \frac{1 - (1 + i)^{-n}}{i} \cdot R \]
\[ R = \frac{P(1 + rt)}{12t} \]
1. DO NOT OPEN THIS EXAM UNTIL INSTRUCTED TO BY YOUR ROOM LEADER. All exam pages must remain stapled. Do not separate or remove any pages. You will have 60 minutes to complete this exam.

2. This exam has 9 pages, including the cover sheet. There are 12 multiple choice questions worth 7 points each and 1 long answer question with six parts worth 16 total points. Only your scantron will be graded for the multiple choice section, what you put on your scantron is your FINAL answer. If you change an answer on the scantron, erase your old answer thoroughly.

3. Blank grids for the multiple choice linear programming problems are provided adjacent to each question. They are provided for convenience only and will not be graded. The graph for the long answer question will be graded.
MULTIPLE CHOICE: Mark your FINAL answers on your scantron.

Questions 1-2: Kyle is choosing between a combination of brands of cat food to feed his cat. He determines that the combination should meet certain minimum nutritional requirements: 36 units of protein, 35 units of fat, and 20 units of taurine per week.

Brand A provides 4 units of protein, 7 units of fat, and 10 units of taurine, and costs $3.50. Brand B provides 9 units of protein, 5 units of fat, 2 units of taurine, and costs $4.00.

Kyle wants to provide for his cat while minimizing his total costs. Let \( x \) be the number of cans of food A that he buys per week and \( y \) be the number of cans of food B.

1. Which of the following is an inequality associated with this problem?
   - a) \( 10x + 2y \leq 20 \)
   - b) \( 4x + 9y \geq 36 \)
   - c) \( 7x - 5y \geq 35 \)
   - d) \( 3.5x + 4y \leq 10 \)
   - e) None of the above

2. Which of the following is one of the vertices of the feasible set?
   - a) (9, 1)
   - b) (0, 4)
   - c) (0, 10)
   - d) (0, 9)
   - e) None of the above
Questions 3-4: An investor has $18,000 to invest in three types of stocks, low-risk, medium-risk and high-risk.

She invests according to three principles. The amount invested in low-risk stocks will be at most $2000 more than the amount invested in medium risk stocks. At least $8000 will be invested in low-risk and medium-risk stocks combined. At least $10,000 will be invested in medium-risk and high-risk stocks combined.

The expected yield for these investments are 4% for low-risk stocks, 5.5% for medium-risk stocks and 7.1% for high-risk stocks. The investor wishes to maximize the yield on the investments. Let x be the amount to be invested in low-risk stocks and y be the amount to be invested in medium-risk stocks.

3. Determine the objective equation for this problem.

- **(a)** Maximize \( Y = 0.04x + 0.055y + 18,000 \)
- **(b)** Maximize \( Y = 1278 - 0.031x - 0.016y \)
- **(c)** Maximize \( Y = 18,000 - 0.031x - 0.016y \)
- **(d)** Maximize \( Y = 1278 + 0.031x + 0.016y \)
- **(e)** None of the above

4. How much should the investor invest in each type of stocks?

- **(a)** $0 low-risk, $8000 medium-risk, $10,000 high-risk
- **(b)** $0 low-risk, $0 medium-risk, $18,000 high-risk
- **(c)** $5000 low-risk, $3000 medium-risk, $10,000 high-risk
- **(d)** $10,000 low-risk, $4000 medium-risk, $4000 high-risk
- **(e)** None of the above
5. If \( n(A) = n(A \cup B) \), which of the following statements about \( A \) and \( B \) must be true?

- (a) \( A \) is a subset of \( B \)
- (b) \( B = A \)
- (c) \( B \) is a subset of \( A \)
- (d) \( B \cap A = \emptyset \)
- (e) None of the above

6. Let \( U = \{a, b, c, d, e, f\} \), \( A = \{d, e, f\} \), \( B = \{a, d, e\} \) and \( C = \{a, c, d\} \).

Which of the following is \((A \cup B)' \cap C\)?

- (a) \( \{a, b, c, d, e, f\} \)
- (b) \( \{a, d, e, f\} \)
- (c) \( \{b, c\} \)
- (d) \( \{b\} \)
- (e) None of the above
7. If \( n(A \cap B) = 42 \), \( n(A) = 100 \) and \( n(B) = 50 \), find \( n(A \cup B) \).

\[
n(A \cup B) =
\]

a) 108  
b) 150  
c) 42  
d) 192  
e) None of the above

8. Which of the following is the shaded region below?

\[
\begin{align*}
\textbf{a)} & \quad R \cap S \cap T \\
\textbf{b)} & \quad (R \cap S) \cup T' \\
\textbf{c)} & \quad (R \cap S) \cap T' \\
\textbf{d)} & \quad R \cup S \cup T \\
\textbf{e)} & \quad (R \cap S)' \cup T
\end{align*}
\]
Questions 9-10: A merchant surveyed 135 people to determine the way they learned about an upcoming sale. The survey showed that 60 learned about the sale from the radio, 80 from television, 30 from the newspaper, 20 from radio and television, 15 from radio and newspapers, 10 from television and newspapers, and 10 from all three sources.

9. How many people learned of the sale from television only?
   a) 60
   b) 80
   c) 25
   d) 10
   e) None of the above

10. How many people learned of the sale from both radio and television?
    a) 140
    b) 20
    c) 100
    d) 0
    e) 10
11. Use De Morgan’s Laws to simplify the expression $(S' \cup T)'$.

   - (a) $S \cup T''$
   - (b) $S \cup T$
   - (c) $S \cap T$
   - (d) $S \cap T''$
   - (e) $\emptyset$

12. Let $U = \{\text{Finite Math students this semester}\}$, $A = \{\text{students who like multiple choice questions}\}$, and $B = \{\text{students who like long answer questions}\}$. Describe the following set using set-theoretic notation:

   \{\text{students who like long answer questions but not multiple choice questions}\}.

   - (a) $A'$
   - (b) $B \cap A$
   - (c) $A' \cap B$
   - (d) $A \cap B'$
   - (e) None of the above
Question 13: A coffee roaster has warehouses in St. Louis and Topeka and receives orders from coffee shops in Kansas City and Columbia. The St. Louis warehouse has 500 lbs of coffee and the Topeka warehouse has 700 lbs of coffee. The Kansas City coffee shop requests 400 lbs and the Columbia store 350 lbs.

The cost of shipping from St. Louis to Kansas City is $4 per pound and St. Louis to Columbia is $2 per pound. The cost of shipping from Topeka to Kansas City is $2.50 per pound and the cost of shipping from Topeka to Columbia is $3 per pound.

Let \((x, y)\) correspond to \(x\) pounds of coffee shipped from St. Louis to Columbia and \(y\) pounds of coffee shipped from St. Louis to Kansas City. The coffee roaster wishes to find the \((x, y)\) that minimizes the company’s shipping costs.

a) Write the set of six inequalities that \((x, y)\) must satisfy.

\[
\begin{align*}
x &\geq 0 \\
y &\geq 0 \\
x &\leq 350 \\
y &\leq 400 \\
x + y &\leq 500 \\
400 - y + 350 - x &\leq 700
\end{align*}
\]

4 points if correct, 1 point each for the last four.

b) Write the objective equation.

\[2050 - x + 1.5y\quad \text{minimize}\]

2 points if correct, 1 point partial credit for minor arithmetic mistakes

c) Use the grid provided to graph the feasible set.

see graph above

3 points if correct, 1 point for bounding rectangle, 1 point for diagonal lines, 1 point for identifying feasible set

d) Identify the vertices of the feasible set.

\[
\left\{ (0, 400), \quad (350, 0), \\
(0, 50), \quad (350, 150), \\
(50, 0), \quad (100, 400) \right\}
\]

3 points if correct, 0.5 points for each vertex
e) Which values for $x$ and $y$ minimize the cost of fulfilling the order?

$(350, 0)$

2 points if correct

f) How much coffee should then be shipped from Topeka to Columbia?

0 pounds

2 points if correct
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1. An experiment consists of flipping a fair coin 6 times and observing the sequence of heads or tails. How many different outcomes have at least 3 heads?

   a) 42
   b) 22
   c) 1
   d) 63
   e) None of the above

2. In downtown Columbia, a student must walk 5 blocks south and 7 blocks east (with no backtracking) to reach her destination, but she wants to stop at Sparky’s along the way as in the picture below. How many different routes are possible?

   a) 792
   b) 300
   c) 400
   d) 3600
   e) None of the above
3. A child is deciding on his favorite and second-favorite crayons from a box. If he can perform this task in 4032 ways, how many crayons are in the box?

- **a** 4032
- **b** 2016
- **c** 64
- **d** 63
- **e** None of the above

4. A class has 20 students of which 6 are seniors. How many different committees with five members can be made if exactly two members of each committee must be a senior?

- **a** 1820
- **b** 15,504
- **c** 65,520
- **d** 5460
- **e** None of the above
5. A number is said to be a palindrome if it reads the same way backwards and forwards (for example, 3453223543). How many ten-digit numbers are palindromes?

- (a) 8,100,000,000
- (b) 9,000
- (c) 100,000
- (d) 90,000
- (e) None of the above

6. An experiment consists of tossing a coin five times and observing the sequence of heads (H) and tails (T). What is the probability of the event "HHHHH"?

- (a) \( \frac{1}{16} \)
- (b) \( \frac{1}{2} \)
- (c) \( \frac{1}{32} \)
- (d) 1
- (e) 0
7. Suppose that \( \Pr(E) = 0.6 \), \( \Pr(F) = 0.7 \) and \( \Pr(E \cap F) = 0.6 \). Find \( \Pr(E \cap F') \).

\[
\begin{array}{ll}
(a) & 0.30 \\
(b) & 0.70 \\
(c) & 0.10 \\
(d) & 0.42 \\
(e) & \text{None of the above}
\end{array}
\]

8. Gamblers usually give odds against an event occurring (the odds of an event not happening). Say a bookie gives 5 to 6 odds against a certain event happening. What does the bookie believe the probability of the event to be?

\[
\begin{array}{ll}
(a) & \frac{6}{11} \\
(b) & \frac{5}{11} \\
(c) & \frac{5}{6} \\
(d) & \frac{6}{5} \\
(e) & \text{None of the above}
\end{array}
\]
9. A box contains 10 computer disks, 2 of which are known to have bad sectors. An inspector removes 3 disks at random from the box and tests them. What is the probability that the inspector discovers at least one disk with bad sectors?

- **(a)** \(\frac{7}{15}\)
- **(b)** \(\frac{3}{10}\)
- **(c)** \(\frac{8}{15}\)
- **(d)** \(\frac{7}{10}\)
- **(e)** None of the above

10. Six people are selected at random. What is the probability that at least two of them were born on the same day of the week? (Answers are rounded to 4 decimal places.)

- **(a)** 1.0000
- **(b)** 0.0428
- **(c)** 0.9572
- **(d)** 0.8571
- **(e)** None of the above
11. A man, a woman, and their four children randomly stand in a row for a family picture. What is the probability that the parents will be standing next to each other? (Answers are rounded to 4 decimal places.)

(a) 0.3333
(b) 0.6667
(c) 0.0417
(d) 0.5000
(e) 0.2500

12. Let $S$ be a sample space and $E$ and $F$ be events associated with $S$. Suppose that $\Pr(E) = 0.4$, $\Pr(F) = 0.3$ and $\Pr(E \cap F) = 0.2$. Calculate $\Pr(F^c \mid E^c)$.

(a) $\frac{5}{14}$
(b) $\frac{1}{2}$
(c) 0
(d) $\frac{5}{6}$
(e) None of the above
13. An urn contains 8 green balls and 5 white balls. A sample of 4 balls is selected without replacement from the urn. Determine the probability that all four balls in the sample are the same color. (Answers are rounded to 4 decimal places.)

- a) 0.0979
- b) 0.0070
- c) 0.1049
- d) 0.8951
- e) None of the above

14. A rancher observes his herd of cattle and determines 45% of them are brown and 55% are black and white. Furthermore, 61% of the cattle that are brown are female and 28% of the cattle that are black and white are female. What is the probability that a randomly selected member of the herd is both black and white and male? (Answers are rounded to 4 decimal places.)

- a) 0.1755
- b) 0.7200
- c) 0.1540
- d) 0.3960
- e) 0.1620
15. Consider a population in which 5 in every 10,000 people have tuberculosis (TB). Suppose there is a skin test available where people with TB test 98% positive and 2% negative, whereas people without TB test positive 1% of the time and negative 99% of the time. Determine the positive predictive value (PPV) of the test, where PPV of a medical test is the probability that a person has TB given that they had a positive result on the test. (Answers are rounded to four decimal places.)

\[
\begin{array}{c} 
\text{a} & 0.0490 \\
\text{b} & 0.0467 \\
\text{c} & 0.9800 \\
\text{d} & 0.0005 \\
\text{e} & \text{None of the above}
\end{array}
\]