1. Sketch the following curve, indicating relative extrema, inflection points, intercepts (if possible), asymptotes (if any), etc.

\[ y = x^4 + \frac{1}{3}x^3 - 2x^2 - x + 1 \]

2. A closed rectangular box with a square base is to be constructed using two different types of wood. The top is made of wood costing $3 per square foot and the remainder is made of wood costing $1 per square foot. If $48 is available to spend, find the dimensions of the box of greatest volume that can be constructed.
   a. Find the equation of the objective function.
   b. Find the constraint equation.
   c. Find the dimensions of the box that maximizes the volume.
   d. Find the maximum volume.

3. A publishing company sells 400,000 copies of a certain book each year. Ordering the entire amount printed at the beginning of the year ties up valuable storage space and capital. However, running off the copies in several partial runs throughout the year results in added costs for setting up each printing run, which costs $1000. The carrying costs, figured on the average number of books in storage, are $0.50 per book. Find the most economical lot size, i.e. the production run size that minimizes the total cost.
   a. Let x be the order quantity and r the number of orders placed during the year. Find the inventory cost in terms of x and r.
   b. Find the constraint function.
   c. Determine the economic order quantity that minimizes the inventory cost.
   d. Find the minimum inventory cost.

4. Let \( f(t) \) be the amount of oxygen (in suitable units) in a lake t days after sewage is dumped into the lake, and suppose that \( f(t) \) is given approximately by the following equation: \[ f(t) = 1 - \frac{6}{t+6} + \frac{36}{(t+6)^2} \]. At what time is the oxygen content increasing the fastest?
5. The demand equation for a certain commodity is given by:

\[ p = \frac{1}{12} x^2 - 8x + 192, \quad 0 \leq x \leq 48. \]

a. Find the value of \( x \) that maximizes revenue, \( R(x) \).
b. Find the corresponding price, \( p \), that maximizes revenue.

6. Until recently, hamburgers at a local sports arena cost $5.70 each and the food concessionaire sold an average of 6500 hamburgers on game night. When the price was raised to $6.40 per hamburger, sales dropped off to an average of 3000 per night.

a. Let \( x \) be the average number of hamburgers sold on a game night and let \( p \) be the price of each hamburger. Assuming a linear demand curve, find the price, \( p \), in terms of the average number of hamburgers sold on a game night.
b. Determine the revenue as a function of the nightly sales \( x \).
c. Find the number of hamburgers that should be sold to maximize revenue.
d. Find the price per hamburger charged to maximize revenue.
e. If the concessionaire has fixed costs of $2500 per night and variable costs of $0.70 per hamburger, what is the price per hamburger that will maximize profit?

7. A rectangular page is to contain 12 square inches of print. The page is to have a 3-inch margin on top and on the bottom, and a 1-inch margin on each side. Find the dimensions of the page that minimize the amount of paper used.

8. Find the derivatives of the following functions:

a. \( y = (x^3 - 1)(2x^2 - x + 1) \)

b. \( f(x) = \frac{3x^2 - 4x + 7}{x^3 + 7x^2 - 6} \)

c. \( f(x) = (4x^5 - 2x^3 + 7x)^{100} \)

d. \( y = \left(\frac{2x}{x + 1}\right)^3 \)

e. \( g(x) = x\sqrt{1 + 4x^2} \)

9. Ecologists estimate that when the population of a certain city is \( x \) thousand persons, the level of carbon monoxide in the air will be \( L \) parts per millions where \( L = 50 + 0.4x + 0.0006x^2 \). The population of the city \( t \) years from now is estimated to be \( x = 752 + 23t + 0.5t^2 \) thousand persons.

a. Find the rate of change of carbon monoxide with respect to the population of the city.
b. Find the time rate of change in population 2 years from now.
c. How fast, with respect to time, will the carbon monoxide level be changing 2 years in the future?
10. Find \( \frac{dy}{dx} \) if \( x \) and \( y \) are related by the equation: \( 3x^5 + y = 3y^5 + x \).

11. Suppose that in Boston the wholesale price \( p \) of oranges (in dollars per crate) and the daily supply \( x \) (in thousands of crates) are related by the equation \( px + 7x + 8p = 328 \). If there are 4 thousand crates available today at a price of $25 per crate, and if the supply is decreasing at the rate of 300 crates per day, at what rate is the price changing?